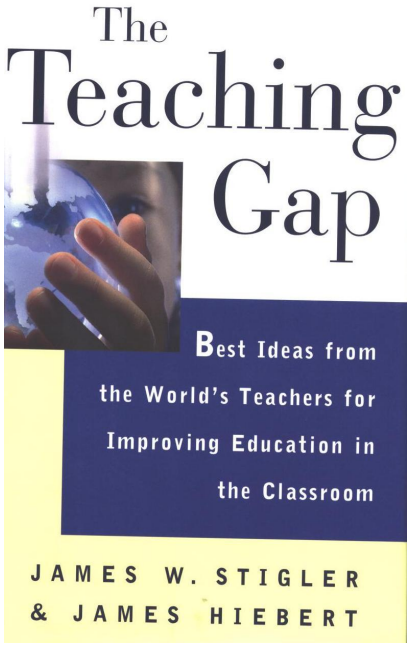
Teaching Through Problem Solving

Session 2



facilitated by
Kathy Kubota-Zarivnij

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Growing Accessible Interactive Networked Supports

- use curriculum expectations and mathematical processes (i.e., grades 6 to 10) to anticipate a range of solutions to a problem, to understand and describe the mathematics in student solutions, and to judge the appropriateness of problems for teaching/learning
- develop a knowledge package or landscape of clustered curriculum expectations
- use the three-part problem solving lesson design to frame the use of problems for teaching mathematics
- develop strategies and mathematics knowledge for anticipating student responses and understanding students' mathematical thinking
- experience strategies for developing students' mathematical communication through the discourse of a math-talk learning community, teacher recording strategies (blackboard writing – mathematical annotations), and coordination and recording of discussion (bansho)
- vary the structure of the problem for students to practise new learning and to provoke use of some strategies and not others, moving towards strategies that can generalize

Learn Mathematics *for* Teaching
2005)

- Figure out why procedures work, not just how to do them
- Try to solve problems in more than one way
- Listen to and probe others' thinking, especially when struggling
- Study students' thinking and work
- Talk in class; practise speaking mathematics

Adolescence is about ...
Dr. Anne Watson (UK)

- identity
- belonging
- being heard
- being in charge
- being supported
- reorganising neural pathways in frontal cortex
- feeling powerful
- understanding the world
- negotiating authority
- arguing in ways which make adults listen

Watson, A. (2007). Raising Achievement in Secondary Mathematics. UK: Open University Press

Benefits of Inquiry and Problem-Solving Tasks

- Choice; action (agency)
- Conjectures; perspectives (identity)
- Ownership (empowerment; identity)
- Discussion (collaboration)
- Reflection
- Changes in their mathematical activity

K Watson, A. (2007). Raising Achievement in Secondary Mathematics. UK: Open University Press

Adolescent Actualisation in Mathematics

- identity as active mathematics thinker
- belonging to the math class community
- being heard by the teacher and classmates
- understanding the world using mat
- negotiating the authority of the teacher through mathematics
- being able to argue mathematically in ways which make adults listen

K Watson, A. (2007). Raising Achievement in Secondary Mathematics. UK: Open University Press

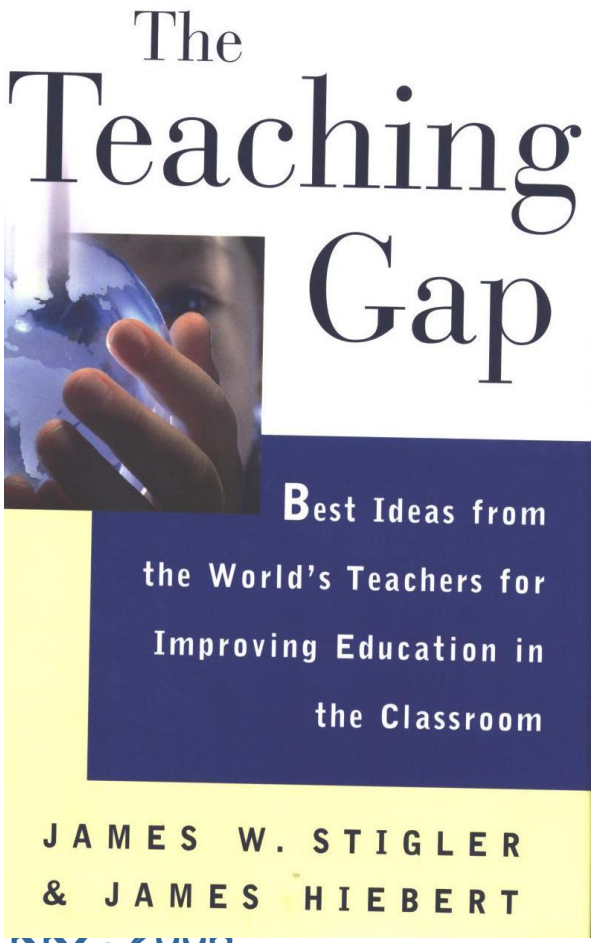
Adolescent Actualisation in Mathematics

- being in charge of personal example space
- being supported by inherent sense of mathematics
- feeling powerful by being able to generate mathematics
- being helped to make explicit shifts of conceptualisation

K Watson, A. (2007). Raising Achievement in Secondary Mathematics. UK: Open University Press

Setting the Context – TIMSS

Understanding the Problem … Achievement Gap



"As important as it is to know how well students are learning, examinations of achievement scores alone can never reveal how the scores might be improved.

We need information on the classroom processes – on teaching – that are contributing to the scores."

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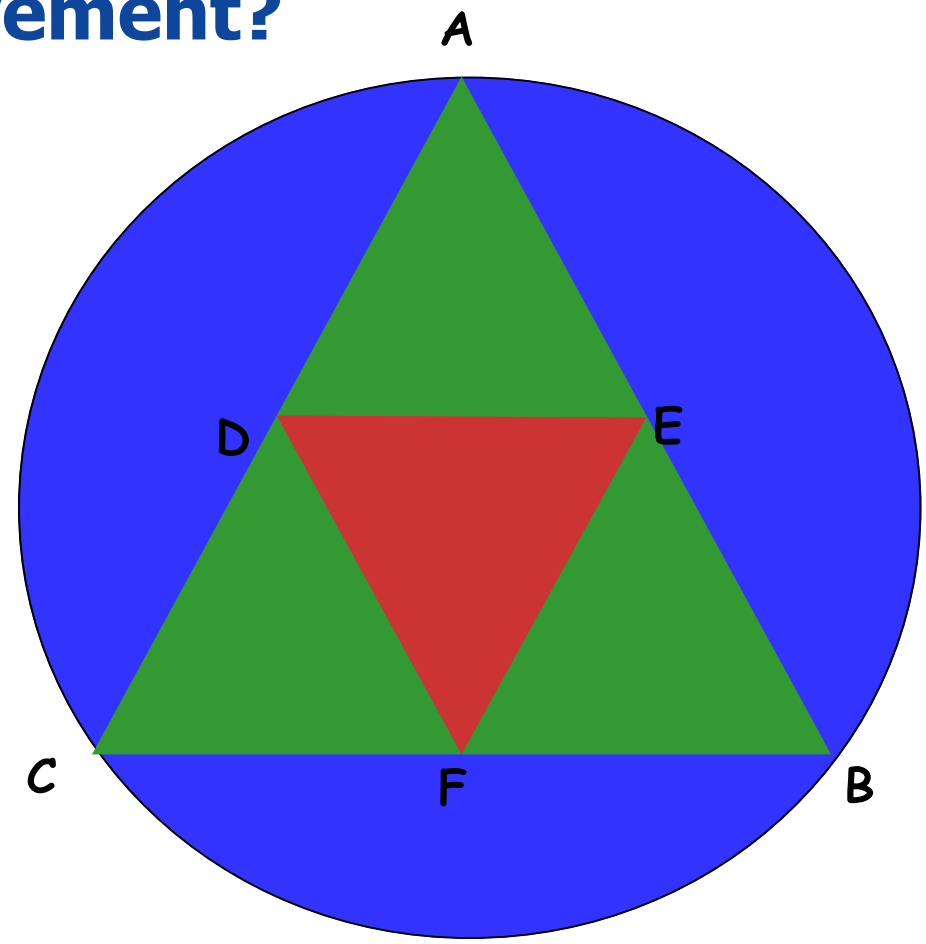
Setting the Context - TIMSS

What if teachers do not know how to improve student learning/achievement?

Instructions:
Describe the properties and the arrangement of the geometric shapes.

BUT apply this condition:
You do NOT know any words that contain the letter "n".



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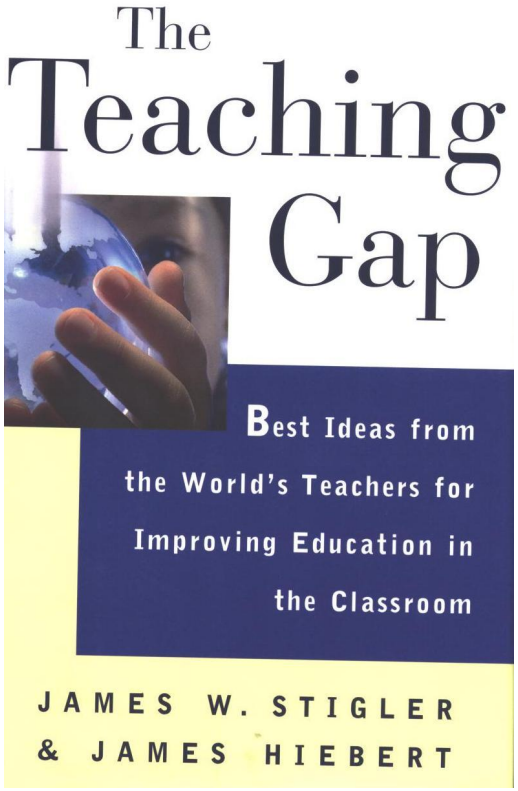
Setting the Context - TIMSS

Teaching is a Cultural Activity



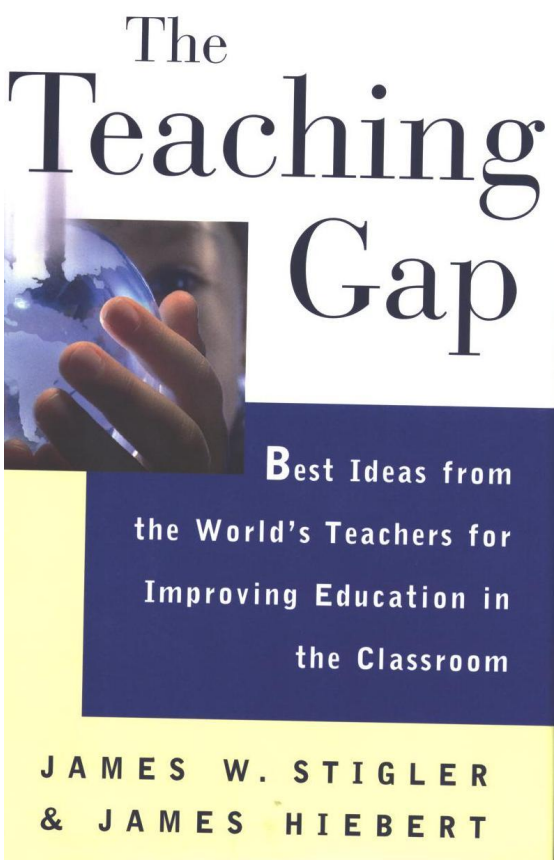
"Teaching is a cultural activity. We learn how to teach directly, through years of participation in classroom life, and we are largely unaware of some of the most widespread attributes of teaching in our own culture."

"If we wish to make wise decisions, we need to know what is going on in they typical classrooms."

"Video information can shake up the way we think and let us take a fresh look at classrooms."

(Stigler & Hiebert, 1999)

Setting the Context - TIMSS
Why U.S., Japan, Germany?

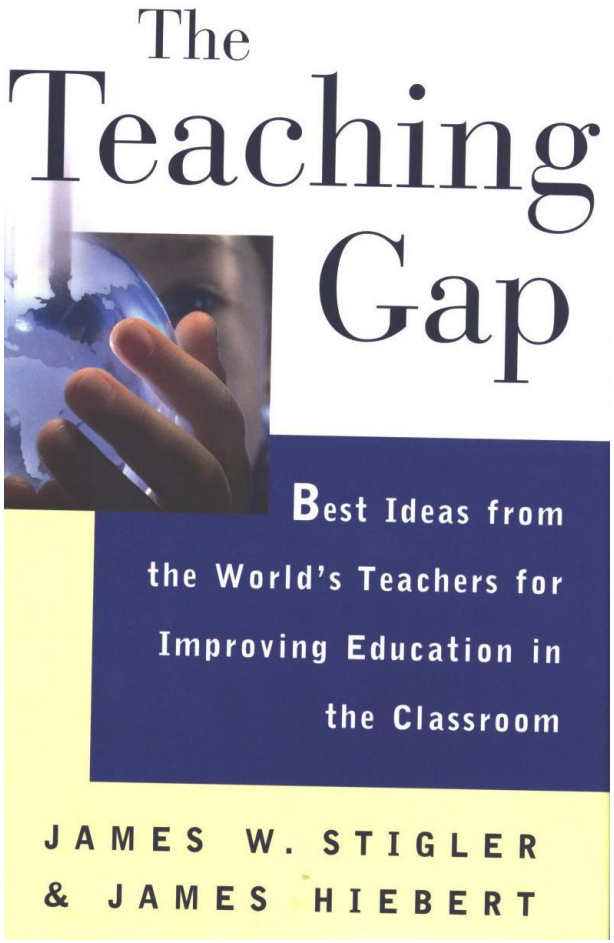


(Stigler & Hiebert, 1999)

- Goal – Through video study, describe and compare 8th grade mathematics instruction in United States, Germany, and Japan
- TIMSS study largely funded by US gov't
- Japan – scored near the top in all international comparisons of mathematics achievement for decades
- Germany – an important comparison country, because like Japan, it is a major economic competitor of United States

Setting the Context - TIMSS

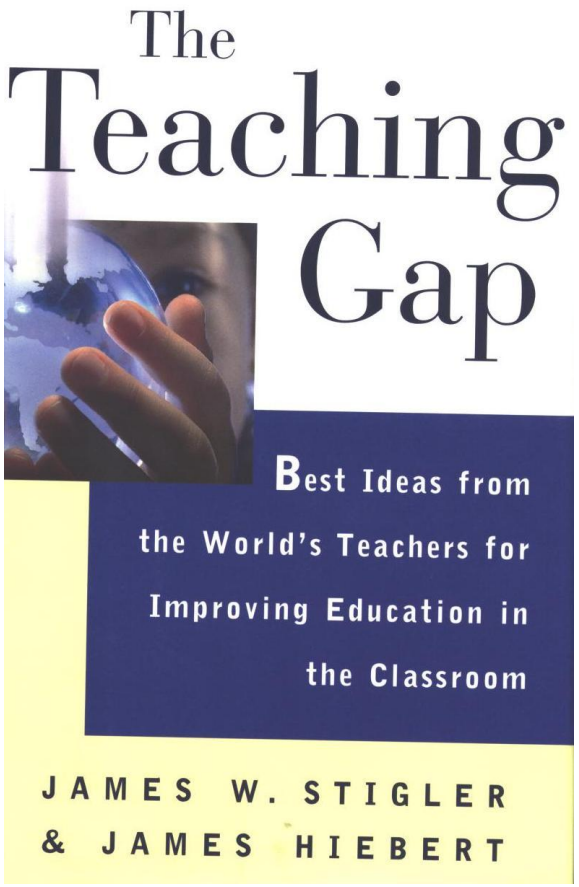
Why Videotaping … How?



(Stigler & Hiebert, 1999)

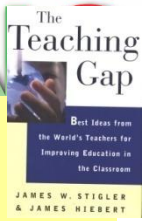
- different teachers use the same words to mean different things
- data collected – random sample of 238 grade 8 mathematics classrooms
  - videotape of a grade 8 lesson
  - teacher questionnaire about their day before and day after teaching plans
  - collection of teaching materials used (textbook pages, worksheets)

Setting the Context - TIMSS
Observation Codes and Analysis



(Stigler & Hiebert, 1999)

- taped a typical lesson scheduled for that teaching day
- to avoid bias – they developed a set of standard observation codes to identify and quantify the frequency of specific teaching events
- a team of 6 coders independently watched the 238 videos and identified images of teaching and assigned observation codes and recorded frequency

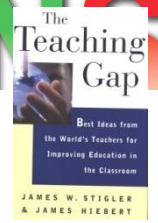


Setting the Context - TIMSS

A mathematician's perspective

(Stigler & Hiebert, 1999)

- "In Japan, there is the mathematics on one hand and the students on the other. The students engage in the mathematics and the teacher mediates the relationship between the two." - ***structured problem solving***
- "In Germany, there is the mathematics as well, but the teacher owns the mathematics and parcels it out to students as she sees fit, giving facts and explanations at just the right time." - ***developing advanced procedures***
- "In the U.S. lessons, there are the students and there is the teacher. I have trouble finding the mathematics. I just see interactions between the teacher and the students." - ***learning terms and practising procedures***

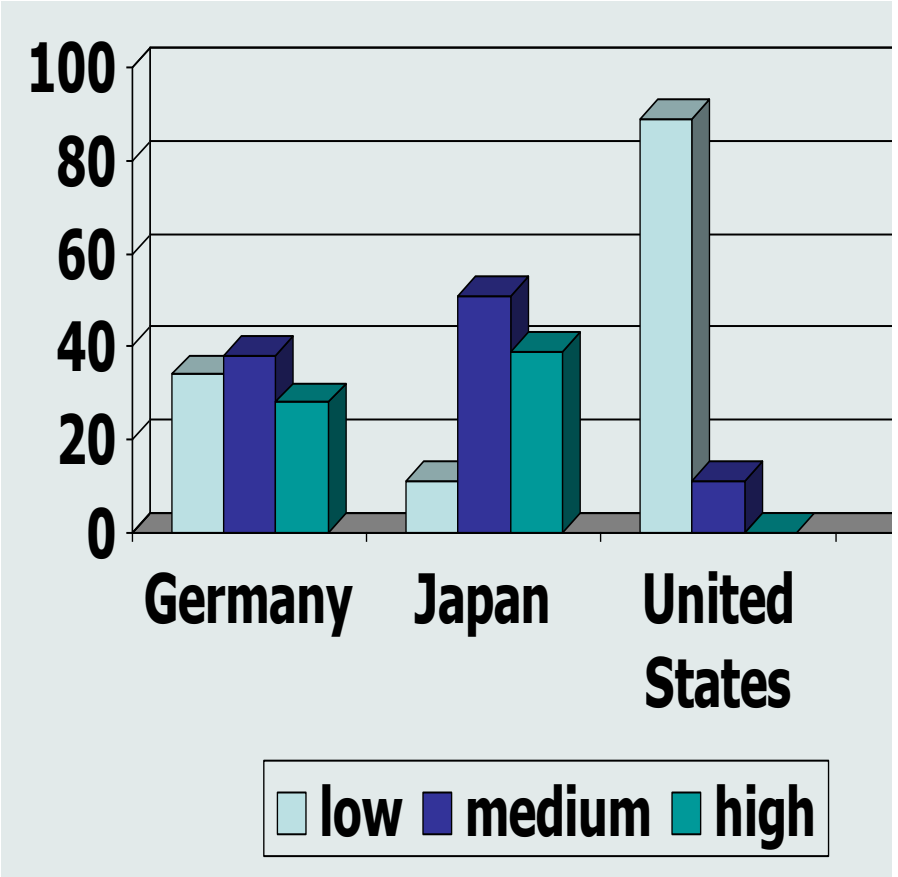


Setting the Context - TIMSS
Mathematics Instruction

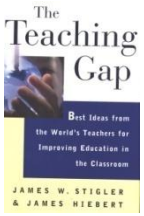
(Stigler & Hiebert, 1999)

Nature of Content

- coded quality of content based on level of challenge and how the content was developed
- definitions vs. rationale and reasoning used to derive understanding



Setting the Context - TIMSS
Mathematics Instruction



Content Elaboration

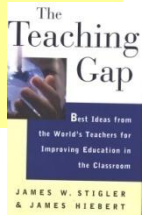
- *stated* concepts *by* teacher and student
- *developed* concepts *through* teacher and student *discussion*

Threats to Coherence

- *switches* in topics
- *interruptions* from external events (e.g., PA announcements never occurred in Japanese lessons, 13% of the time in German lessons and 31% of the time in U.S. lessons)

(Stigler & Hiebert, 1999)

Setting the Context - TIMSS
Mathematics Instruction



Content Coherence

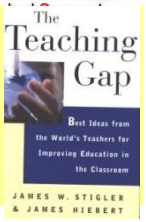
- connectedness or relatedness of the mathematics *across the lesson* (e.g., a well-formed story consists of a sequence of events that fit together to reach a final conclusion)

Making Connections

- weaving together ideas and activities in the relationships *between the learning goal and the lesson task* made explicit by teachers
  - 92% of Japanese teachers
  - 76% of Germany teachers
  - 45% of U.S. teachers

(Stigler & Hiebert, 1999)

Setting the Context - TIMSS
Engaging Students in Mathematics



Who Does the Work?

- who *controls the solution method* to a problem predominately student controlled
  - 9% of the time in the U.S.
  - 9% in Germany
  - 40% of the time in Japan classes

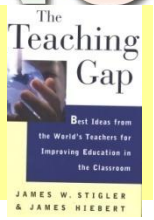
What Kind of Work is Expected?

- German and U.S. students spent *most* of their *time practising routine procedures*
- Japanese students spend *equal time practising procedures* and *inventing new methods*

(Stigler & Hiebert, 1999)

Setting the Context - TIMSS

Beliefs about Mathematics Teaching



Nature of Mathematics

- U.S. (math is a *set of procedures* and skills)
- Japan (math is about *seeing new relationships* between mathematical ideas)

Nature of Learning

- U.S. students become *proficient executors of procedures*
- Japanese students learn by
  - first *struggling to solve* math problems
  - then *participating in discussions* about how to solve them hearing pros and cons, *constructing connections* between methods and problems
  - so they use their time to *explore*, *invent*, *make mistakes*, *reflect*, and *receive needed information* just in time

(Stigler & Hiebert, 1999)

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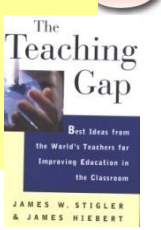
Setting the Context - TIMSS

- Problem-Solving Lesson Design



BEFORE

- Activating prior knowledge; discussing previous days' methods to solve a current day problem

DURING

- Presenting and understanding the lesson problem
- Students working individually or in groups to solve a problem
- Students discussing solution methods

AFTER

- Teacher coordinating discussion of the methods (accuracy, efficiency, generalizability)
- teacher highlighting and summarizing key points
- Individual student practice

(Stigler & Hiebert, 1999)

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Problematic Aspects of Intermediate Mathematics

- probability
- proportion & ratio
- non-linear sequences
- symbolic representation
- proving things
- adding fractions…..
- understanding limits
- using algebraic relationships
- reasoning from properties …

K Watson, A. (2007). Raising Achievement in Secondary Mathematics. UK: Open University Press

Three-Part Problem Solving Lesson Design

Before (Activating Knowledge)

Work in groups of 3.

1. Write down two numbers.
2. Write down another two numbers which total 36.
3. Write down a third set of two numbers with a difference of 8.
4. Write down a fourth set of two numbers with a total of 36 and a difference of 8.

Focus Observations on:

- drawn, modeled, and written mathematical work
- learning actions and interactions
- oral, modeled, and written evidence of learning

How could you adjust your number pairs to meet criterion #4?

Three-Part Problem Solving Lesson Design

During (Working On It) – Bus Problem

There are 36 children on school bus.

There are 8 more boys than girls.

How many boys? How many girls?

a) Solve this problem in 2 different ways.

b) Show your work.

What details from the Bus Problem will you use to make a plan to solve this problem?

Polya's Problem Solving Process – Understand the problem, Make a Plan

Three-Part Problem Solving Lesson Design

After (Consolidation) – Bus Problem

There are 36 children on school bus.

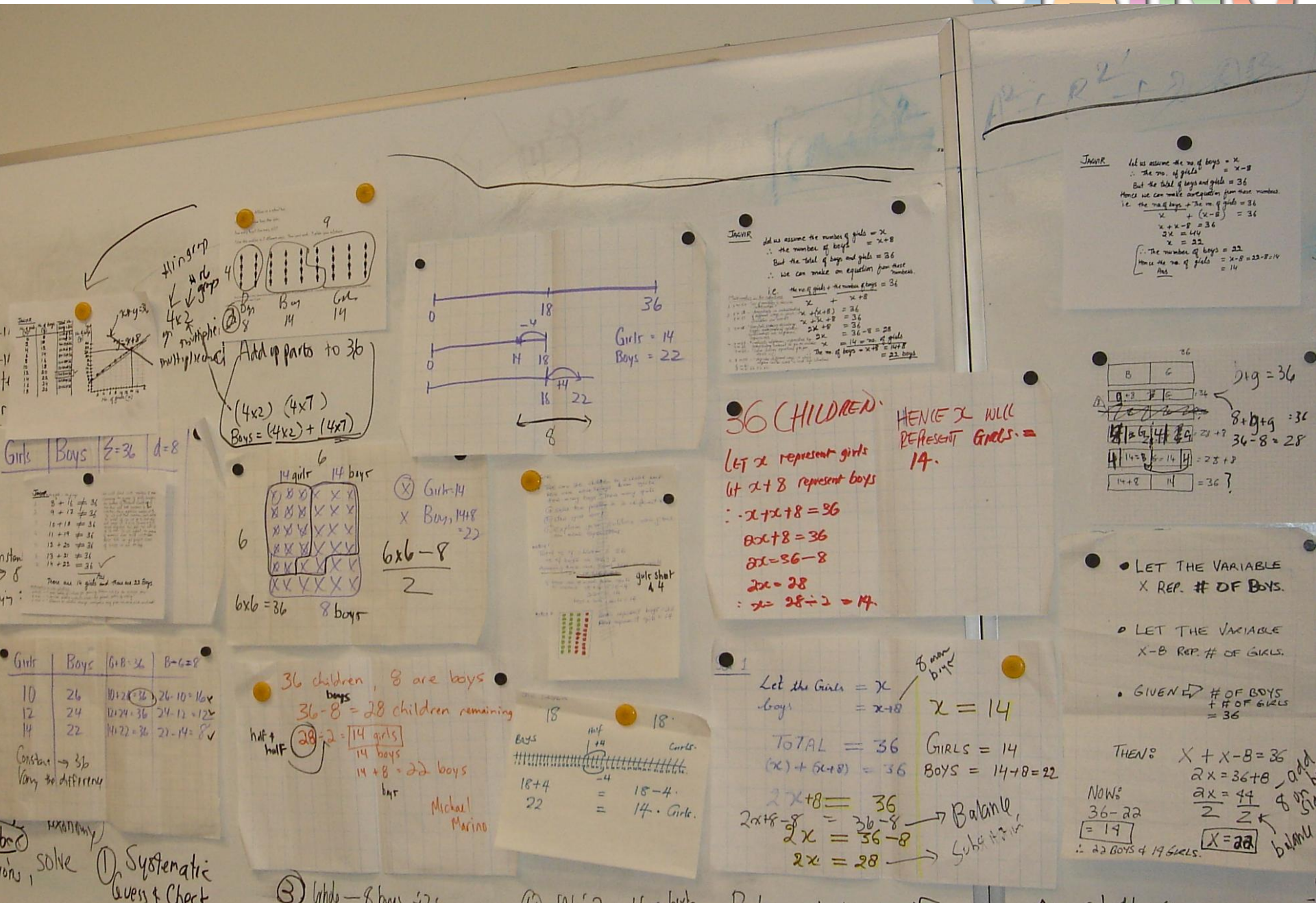
There are 8 more boys than girls.

How many boys? How many girls?

a) Solve this problem in 2 different ways.

b) Show your work.

- How are our solutions similar and different?
- What mathematics (curriculum expectations) are demonstrated in the solutions?
- What are the mathematical relationships between the solutions?


Girls | Boys | Σ=36 | d=8
Add up parts to 36
(4x2) (4x7)
Boys = (4x2) + (4x7)
Girls = 14
Boys = 22
36 CHILDREN
Let x represent girls
& x+8 represent boys
HENCE x WILL REPRESENT GIRLS = 14.
6x6 − 8 / 2
6x6 = 36
8 boys
36 children, 8 are boys
36 − 8 = 28 children remaining
half & half
14 girls
14 boys
14 + 8 = 22 boys
Michael Mariao
Let the Girls = x
boys = x+8
TOTAL = 36
(x) + (x+8) = 36
2x+8 = 36
2x+8−8 = 36−8
2x = 36−8
2x = 28
x = 14
GIRLS = 14
BOYS = 14+8 = 22
Balance
Solve it!
LET THE VARIABLE X REP. # OF BOYS.
LET THE VARIABLE X−8 REP. # OF GIRLS.
GIVEN ⇨ # OF BOYS + # OF GIRLS = 36
THEN: X + X − 8 = 36
2x = 36+8
2x/2 = 44/2
X = 22
NOW: 36 − 22 = 14
∴ 22 BOYS & 14 GIRLS.
solve
1. Systematic Guess & Check
3. Make a table